Calculus I

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Practice Second Exam

Remember remember the 5th of November, 2014

Problem	Possible points	Score
1	20	
2	20	
3	20	
4	10	
5	10	
6	20	
Total	100	

Problem 1. (20p) Find the absolute maximum and minimum of $f(x) = x^3 - x^2 - x + 2$ on $[0, 2]$.

Problem 2. (20=10+10p) Find the limits

(a) $\displaystyle\lim_{x\to 1} \frac{\ln x}{\tan \pi x}$

(b) $\displaystyle\lim_{x \to 0+} x^{\cos x - 1}$

Problem 3. (20=10+10p)

(a) Find the inverse of $F(t) = 13e^{5t}$.

(b) Show that the function $f(x) = 2x + \cos x$ is invertible and find $(f^{-1})'(1)$.

Problem 4. (10=5+5p) Using Newton's method for the approximation of $\sqrt[3]{7}$ regarded as a zero of the function $f(x) = x^3 - 7$:

(a) Find the general formula for x_{n+1} in terms of x_n.

(b) Find x_2 if $x_1 = 2$.

Problem 5. (10p) A rectangle has two corners on the x-axis and the other two on the parabola $y = 12 - x^2$, with $y \geq 0$. What are the dimensions of the rectangle of this type with maximum area?

Exercise 6. (20p) Let $f(x) = \dfrac{x}{(x+1)^2}$.

(a) Find points of intersection of the graph with the x-axis and with the y-axis.
(b) Find vertical and horizontal asymptotes.
(c) Find intervals where the function is increasing/decreasing.
(d) Find maxima and minima.
(d) Find intervals where the function is concave up/down.
(e) Find inflection points.
(f) Use information from (a)-(e) to sketch the graph of the function. Clearly indicate the information obtained in (a)-(f) on the graph.